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Micromechanics of Flaw Growth in Static Fatigue: Influence of Residual Contact Stresses

P. CHANTIKUL, B. R. LAWN,* and D. B. MARSHALL*,*

Department of Applied Physics, School of Physics, University of New South Wales, Kensington, New South Wales 2033, Australia

Residual contact stresses about indentation flaws are demonstrated to have a strong deleterious effect on specimen lifetime in static fatigue. The underlying basis of conventional fatigue analysis is first examined critically and is argued to be deficient in the way the stress intensity factor for the flaws is related to the characteristic parameters of crack geometry and applied loading. In general, it is necessary to incorporate a residual term into the stress intensity formulation. A modified theory of static fatigue is accordingly developed, in which the residual contact stresses play a far from secondary role in the micromechanics of flaw evolution to failure. Strength tests on Vickers-indented soda-lime glass disks in water environment provide clear experimental confirmation of the major theoretical predictions. Implications of the residual stress effect concerning fracture mechanics predictions of lifetimes for "real" ceramic components under service conditions are discussed.

I. Introduction

FRACTURE mechanics concepts have been used with considerable success in the design of ceramic components for structural applications.[1-7] Under fatigue conditions, i.e. where strength tends to diminish as the duration of loading is increased, the essential assumption which underlies the fracture mechanics approach is that failure occurs via "slow" growth from initial flaw to critical microcrack. Analysis of the strength characteristics then proceeds in accordance with two basic crack propagation equations: (i) a stress intensity factor equation of the form $K \propto \sigma c^{1/2}$, representing the driving force on the crack of characteristic dimension c subjected to a uniform applied stress σ; (ii) a crack-velocity equation of the form $v=v(K)$, representing the rate of extension at any point in the evolution. With such an analysis it is possible, in principle, to predict service lifetimes of components at any specified stress level. All that is needed is some controlled, short-term crack-growth test to determine characteristic fracture parameters for the particular material/environment system in question.

With the fracture mechanics approach apparently headed for widespread adoption as a vital element in ceramics evaluation programs, it is necessary to be conscious of its limitations. In making this point in their review article on fatigue testing, Wiederhorn and Ritter[7] emphasized the particular need for caution in extrapolating short-term data to extended-duration conditions. Quite apart from the usual difficulties associated with optimization of data processing (e.g. random error propagation, time cost of data collection in low-velocity regions), there is the issue of the universality of the assumed crack-propagation formulas. Wiederhorn and Ritter focused their attention on the crack-velocity equation; they noted the empirical base of the equations in common usage and demonstrated that different functional forms which appear to fit the data equally well over the range of test values can diverge markedly in the regions of extrapolation. In this context, some doubt exists as to the applicability of crack-velocity relations obtained from macroscopic fracture data to the flaws which actually control strength. However, the other basic crack-propagation equation in the fatigue formulation, that which expresses the stress intensity factor in terms of loading and crack-size parameters, was subjected to no such critical examination. Indeed, it is a reflection on the unquestioned acceptance of the elementary form of the stress intensity factor that explicit mention of this essential facet in the theory tends now to be omitted altogether from surveys of fatigue analysis.

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*Member, the American Ceramic Society.
*Now with the Materials and Molecular Research Division, Lawrence Berkeley Laboratories, University of California, Berkeley, California 94720.

Recent investigations into the nature of indentation-induced crack patterns in brittle materials have shown that the simple stress intensity factor relation $K \propto \sigma c^{1/2}$ does in fact require certain qualification.[8-13] In many ceramics, strength is controlled by flaws associated with past surface contact events (e.g. incidental sharp-particle impacts, abrasion damage). The contact deformation generally includes an irreversible component,[9] which becomes manifest as a residual crack driving force in the subsequent failure mechanics.[10] Accordingly, the pertinent expression for the stress intensity factor for microcracks in tensile loading should strictly contain an extra, residual term. The special relevance of this additional factor to fatigue was clearly demonstrated in comparative strength tests at constant stress rate on Vickers-indented glass disks in water; as-indented specimens were systematically weaker than control specimens subjected to a post-indentation anneal, the more so at lower stress rates.[12] There it was shown that use of conventional dynamic fatigue theory (i.e. on the assumption of zero residual stress term) to evaluate basic fracture mechanics parameters could lead to significant discrepancies in the inferred response of well-defined macroscopic cracks. Similar residual-stress effects were observed on glass surfaces damaged by abrasion with irregular particles, indicating a capacity for indentation crack systems to represent fracture behavior at the flaw level.[13]

In this paper the static fatigue characteristics of the same Vickers-indented-glass/water-environment system as used in the previous constant-stress-rate study[12] are investigated, with particular attention to implications concerning the reliability of fracture mechanics in predicting lifetimes.

II. Background Theory

A schematic diagram of the proposed model indentation-flaw system is given in Fig. 1. In stage (A), a Vickers pyramid indenter, peak load P, generates a well-defined "radial/median" crack system,[11] characteristic dimension c. Generally, a second, "lateral" crack system is also produced, but plays only a secondary role in strength determination[12] and is accordingly omitted in the diagram. In stage (B), a tensile stress σ_a is subsequently applied to the crack system. The stress intensity factor for this essentially penny-like crack configuration is[8,10,12]

$$K=\chi_r P/c^{3/2}+\sigma_a(\pi\Omega c)^{1/2} \qquad (c>c_0') \tag{1}$$

The first term on the right represents the residual driving force; χ_r is an appropriate elastic/plastic field parameter, related to the material hardness and elastic modulus, which determines the intensity of the residual field.[11] The second term represents the conventional uniform tensile field contribution, with Ω a crack-geometry parameter. Here c_0' is the crack size just prior to application of the stress σ_a; if the system is exposed to a reactive environment between stages (A) and (B), c_0' will be in excess of the equilibrium value c_0 which would prevail if no slow crack growth were to occur under the action of the residual indentation field alone, i.e. at $\sigma_a=0$, $c=c_0$ for $K=K_c$, where K_c is the toughness. It is interesting to note in Eq. (1) that the residual term will dominate at small c, the very region which is generally assumed to be rate-controlling in the failure kinetics.

Now if the crack configuration in Fig. 1 is subject to subcritical growth conditions during application of a sustained stress σ_a, the time to failure will be determined by an appropriate crack-velocity function. Following our previous course,[12] we adopt the simple power-law function

$$v=v_0(K/K_c)^n \qquad (K<K_c) \tag{2}$$

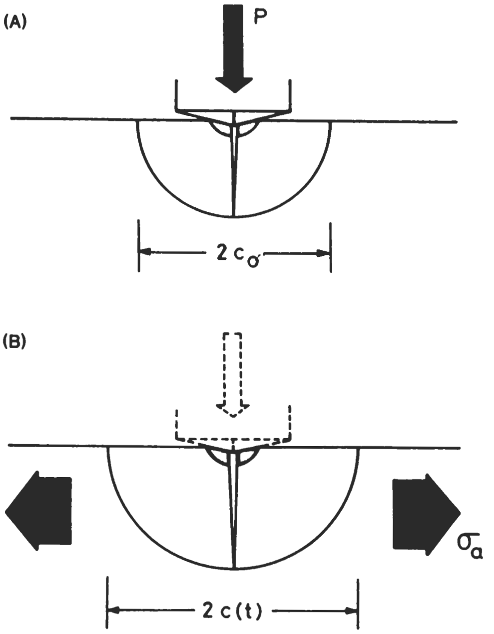

Fig. 1. Model indentation-flaw system. *(A)* Vickers indenter, peak load P, generates radial/median crack, characteristic dimension c (equilibrium value c_0 at completion of contact, with further, subcritical extension to c_0' on exposure to reactive environment). *(B)* Constant tensile stress σ_a augments residual contact field (represented by "ghost" contact), causing crack system to expand subcritically to failure configuration.

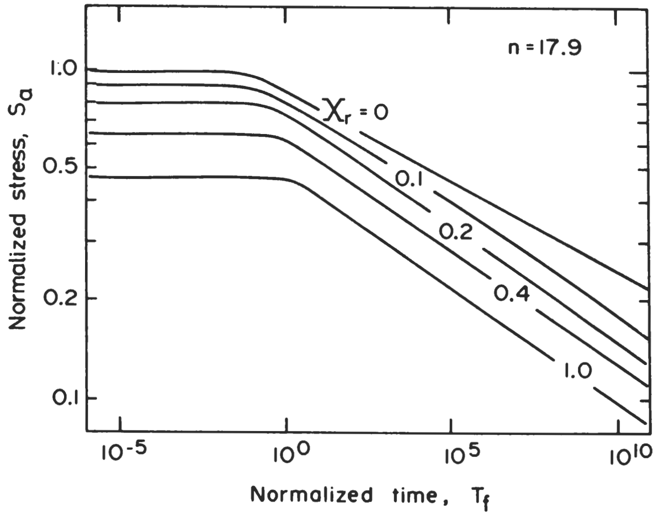

Fig. 2. Normalized static fatigue curves for flaws with different degrees of residual stress, as characterized by parameter X_r; evaluated for index n appropriate to soda-lime-glass/water-environment system.

where v_0 and n are kinetic parameters for the given material/environment system.

Equations (1) and (2) combine to give a differential equation in crack size and time,

$$\dot{c}/v_0=\{[\chi_r/K_c]P/c^{3/2}+[(\pi\Omega)^{1/2}/K_c]\sigma_a c^{1/2}\}^n \tag{3}$$

Given the material/environment parameters, this differential equation must be solved for the time to failure t_f, i.e. the time to expand the crack from its initial dimension (c_0') to its final critical dimension (solution of Eq. (1) at $K=K_c, dK/dc>0$ branch), at each prescribed stress σ_a. In practice, this involves a numerical, stepwise integration procedure.

As in the earlier dynamic fatigue study, for illustrating the general form of the solutions of Eq. (3) it is convenient to normalize the variables.[12] Thus, writing $C=c/c_0'$, $T=tv_0/c_0'$, $X_r=\chi_r P/K_c c_0'^{3/2}$, $S_a=\sigma_a(\pi\Omega c_0')^{1/2}/K_c$, Eq. (3) reduces to

$$\dot{C}=(X_r/C^{3/2}+S_a C^{1/2})^n \tag{4}$$

with $\dot{C}=dC/dT$. The corresponding initial and final conditions for the integration, respectively, become $C=1$ at $T=0$ and $\dot{C}=1$ ($K=K_c$ in Eq. (2)) at $T=T_f$. Figure 2 shows the static fatigue characteristics $S_a(T_f)$ thus evaluated for $n=17.9$ (previously determined coefficient for the particular glass/water system studied here[12]) and for several values of the residual-stress parameter X_r. Note that this latter parameter must lie within the range $0\leq X_r\leq 1$, corresponding to bounds $K=0$ and $K=K_c$ at the zero applied stress configuration $c=c_0'$, $\sigma_a=0$ in Eq. (1). The existence of residual contact stresses would appear to be capable of reducing lifetimes at any stress level by several orders of magnitude.

III. Experimental Procedure

The experimental procedure used in the dynamic fatigue study[12] was closely paralleled here, to allow for a proper comparison of results. Disks (nominally 50 mm diam. by 3 mm thick) were cut from commercial soda-lime glass sheet. Each disk was indented at its center with a Vickers diamond pyramid at peak load $P=5$ N (standard 15 s duration). Polarized-light transmission microscopy was used to examine the level of stress birefringence associated with the residual contact field and to monitor any post-indentation crack development. Accordingly, the radial cracks were allowed to expand in air environment for 30 min, corresponding to a rapidly decelerating growth from $c_0=43\pm3$ μm to $c_0'=67\pm5$ μm. Some of the disks were tested immediately after the 30 min air exposure ("as-indented" specimens); the others were first heat-treated until all stress birefringence disappeared (520°C for 2 days), thereby annealing out the residual contact stresses ("annealed" specimens). In the latter case no additional crack growth was incurred during the heating process itself but a post-anneal stress cycle used to "resharpen" the cracks[12] necessarily produced a small increase, <5 μm, in c_0'.

The strength tests were run in center-symmetric flexure, indented face on the tensile side, in a specially designed fixture.[14] Immediately prior to application of the stress, a drop of distilled water was placed onto the indentation site. (For long-term tests it was necessary to cover the wet area with a thin plastic sheet, to prevent evaporation.) Standard formulas for plates in flexure were used to compute the tensile stress on the cracks from the load applied to the support rings.

IV. Results

The results of tests on as-indented and annealed disks are shown in Fig. 3. Each data point represents the mean and standard deviation (5 to 10 specimens) in logarithmic time to failure at each stress level. The solid lines are predictions from the theory in Section II, using the following "calibration" parameters from the sister study in Ref. 12 in Eq. (3): $K_c/(\pi\Omega)^{1/2}=0.78\pm0.02$ MPa·m$^{1/2}$ (from inert-strength tests on annealed disks); $K_c/\chi_r=31\pm4$ MPa·m$^{1/2}$ for as-indented disks (inert-strength tests), $X_r=0$ for annealed disks; $n=17.9\pm0.5$ and $v_0=2.4\pm0.6$ mm·s^{-1} (dynamic fatigue tests for annealed indentations). With due allowance for the random errors, theory and experiment show general agreement. The systematic difference between the results for the as-indented and the annealed specimens indicates the magnitude of the residual-stress effect.

V. Discussion

Given the existence of a residual driving force term in the mechanics of crack growth for contact-related flaw systems, it is pertinent to examine any ensuing implications concerning the design of long-lifetime components. This may be conveniently done via Fig. 4, in which the three curves are alternative representations of Eq. (3) for as-indented glass disks, i.e. for disks containing flaws

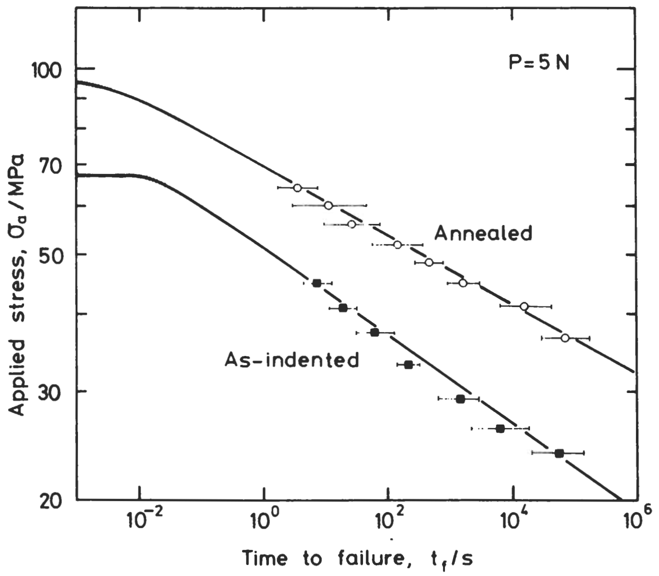

Fig. 3. Static fatigue results for Vickers-indented soda-lime glass disks broken in water environment. Data points are experimental results, solid curves predictions from the indentation theory; shaded regions indicate inert strength levels. Note significantly shorter lifetimes of as-indented disks compared to annealed disks, indicating a strong residual-stress effect.

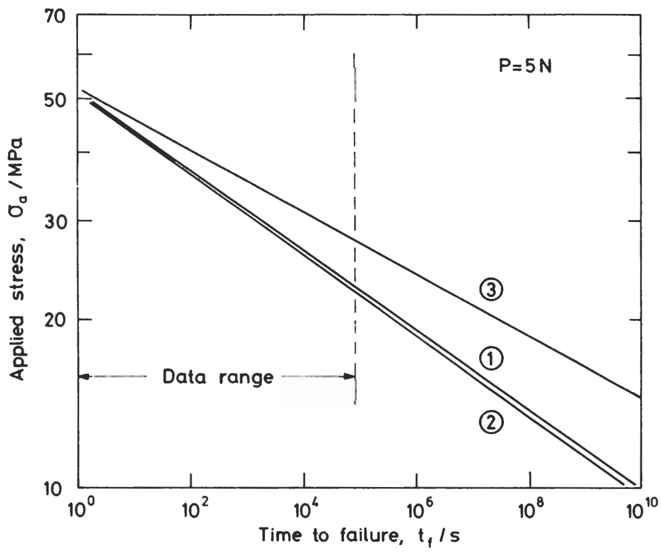

Fig. 4. Predicted static fatigue response of indentation flaws for soda-lime-glass/water-environment system, evaluated from Eq. (3) in accordance with following parameters: (1) nonzero residual stress term, $\chi_r\neq 0$, "true" kinetic parameters n and v_0; (2) zero residual stress term, "apparent" kinetic parameters, "true" initial flaw size; (3) zero residual stress, "true" kinetic parameters, "apparent" initial flaw size. Curves are extrapolated into long-lifetime region beyond data range used in parameter calibrations.

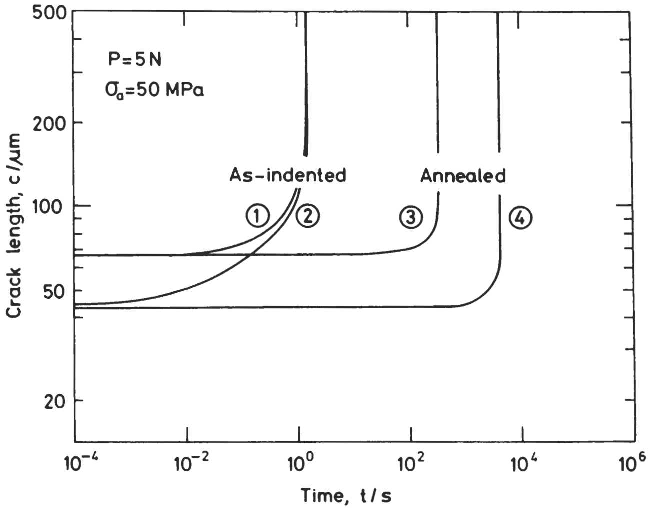

Fig. 5. Predicted time evolution of indentation flaws for soda-lime-glass/water-environment system at specified loading conditions. From Eq. (3), using (1) $\chi_r\neq 0$, $c_0'=67$ µm; (2) $\chi_r\neq 0$, $c_0'=43$ µm; (3) $\chi_r=0$, $c_0'=67$ µm; (4) $\chi_r=0$, $c_0'=43$ µm.

in their "natural" state. Curve 1 is simply an extrapolated version of the lower curve shown in Fig. 3, using the calibration parameters quoted from the earlier dynamic fatigue study. Curves 2 and 3 are representations based on the assumed absence of residual stress, using the standard static fatigue formulation[7] (solution of Eq. 3 at $\chi_r=0$)

$$t_f=B_1/\sigma_a^n=B_2(\sigma_i^0)^{n-2}/\sigma_a^n \tag{5}$$

where $B_1=[K_c/(\pi\Omega)^{1/2}]^n/(n/2-1)v_0(c_0')^{n/2-1}$, $B_2=[K_c/(\pi\Omega)^{1/2}]^2/(n/2-1)v_0$, and $\sigma_i^0=[K_c/(\pi\Omega)^{1/2}]/c_0'^{1/2}$ is the inert strength; curve 2 is generated using "apparent" kinetic parameters $n=13.7$ and $B_1=2.7\times10^{23}$ MPa$^n\cdot$s (which, for the measured initial crack length $c_0'=67$ µm, corresponds to $v_0=55$ mm$\cdot$s^{-1}) evaluated directly from slope and intercept of a linear force fit to dynamic fatigue data for as-indented disks[12]; curve 3 is generated using the "true" kinetic parameters $n=17.9$ and $v_0=2.4$ mm$\cdot$s^{-1} evaluated from dynamic fatigue data on annealed disks[12] to give $B_2=32$ MPa$^2\cdot$s, but in conjunction with the inert strength $\sigma_i^0=68.0$ MPa (corresponding to an "apparent" initial flaw size $c_0'=130$ µm) measured for as-indented disks. Insofar as the "true" values of n and v_0 obtained from annealed indentation cracks may indeed be taken as representative of well-developed cracks in the soda-lime-glass/water-environment system, curve 1 is the prediction of the fracture mechanics theory with the residual term included, using macroscopic crack data. Curve 2 is the prediction of conventional theory, but with fatigue strength data on the same indentation flaws as those which control the lifetime now taken as a base for parameter calibration. Curve 3 is the conventional prediction with inert strength data on the same flaws, in conjunction with macroscopic kinetic parameters, forming the calibration base. Clearly, use of the standard static fatigue formulas can lead to significant errors in the prediction of times to failure. However, these errors are not so critical for curve 2, from which it may be concluded that, if one does choose to ignore the residual stress term in the fracture mechanics, apparent kinetic parameters obtained from fatigue strength data rather than from macroscopic crack velocity data are to be preferred in lifetime evaluation. In this context it must be emphasized that any attempt to bypass the more general solutions of Eq. (3) is made at the expense of a proper understanding of the flaw micromechanics.

In discussing Eq. (1), it was pointed out that the residual term dominates at small crack size. The function $K(c)$ at fixed P and σ_a is thus no longer monotonically increasing but instead passes through a minimum (see Fig. 2, Ref. 12). Consequently, the region of slowest growth under fatigue conditions occurs somewhere between the initial and final configurations; i.e. the initial crack size does not emerge as a rate-controlling parameter, as it does in the usual strength theories. To illustrate this point, the time evolution of crack size as predicted from Eq. (3) is plotted for four cases, at $P=5$ N and $\sigma_a=50$ MPa, in Fig. 5: curves 1 and 2 are for as-indented surfaces, 3 and 4 for annealed surfaces; curves 1 and 3 correspond to a starting crack size $c_0'=67$ µm (i.e. subject to subcritical extension prior to fatigue test), curves 2 and 4 to a starting crack size $c_0'=c_0=43$ µm (zero prior subcritical extension). The influence of the residual contact stresses is thereby exemplified by the way curves 1 and 2 tend to similar failure times, in direct contrast to the divergent response of curves 3 and 4 for cracks in the "standard" state $\chi_r=0$.

It is to be emphasized that the residual stresses discussed in this study refer exclusively to those produced locally by the flaw-producing contact process itself, not to those that might be present in the glass surface prior to indentation, e.g. due to thermal tempering. In principle, it is a straightforward matter to extend the analysis to the more general case, simply by replacing σ_a in Eq. (1) (and thence in Eq. (3)) by an "effective" applied stress with the preexisting stress subtracted. The basis for such an extension of the formulation is given in Ref. 10.

The generality of the residual stress concept discussed in this work may extend beyond the well-defined indentation crack system. Any flaw with a contact-related history, e.g. abrasion flaws,[13] would be expected to show similar effects, although possibly to a lesser extent in severe cases where physical removal of material immediately surrounding the deformation zone occurs (especially in machining damage). Indeed, since the primary cause of the residual stress field lies in the elastic/plastic mismatch strain associated with the local hardness impression,[9,11] analogous behavior might be observed at any defect center whose origins involve some precursor plasticity.

VI. Conclusions

(1) The stress intensity factor for flaws produced by indentation should incorporate a residual contact term.

(2) The indentation-flaw formulation for static fatigue with residual stress term included predicts lower strengths, and correspondingly shorter lifetimes, than conventional fatigue theory. It also predicts relatively little sensitivity of strength characteristics to initial crack size, contrary to usual expectations.

(3) Lifetime predictions made on the assumption of zero residual stress terms in the indentation-flaw analysis can diverge significantly from observed fatigue behavior. The error is likely to be less serious when "apparent" kinetic parameters obtained from strength data are used instead of "true" parameters derived from macroscopic crack velocity data.

(4) The indentation formulation as reflected in the "universal" curves of Fig. 2 has a certain generality, in that it should apply to all ceramics which show susceptibility to subcritical crack growth, and that it should be extendable to components whose surfaces are in a state of prestress. Moreover, the idealized indentation flaws considered here are expected to be representative, to a greater or lesser extent, of naturally occurring flaws.

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